Cogging Torque Minimization Design in Interior Permanent Magnet Motor by Using an Analytical Method

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This paper presents a new magnetic equivalent circuit (MEC) model for calculating the cogging torque in an interior permanent magnet synchronous motors (IPMSMs). Since the proposed method can take into account the slotting effect and magnet end effect, the analysis and design of IPMSMs can be conducted correctly and rapidly despite no use of a finite element method (FEM). Its validity was confirmed by comparing the results of the proposed method with those of FEM. Furthermore, the optimal design result considering various objectives is elicited by using the proposed method.

***Index Terms*— Analytic method, conformal mapping, interior permanent magnet synchronous motor, magnetic equivalent circuit.**

I. Introduction

DUE to their high torque density and efficiency, interior permanent magnet synchronous motors (IPMSMs) are widely used for many industrial applications [1]. However, the IPMSMs have a complex structure and non-linear characteristics, which make it difficult to analyze them by using analytical method. Although a finite element method (FEM) can precisely obtain the flux density, it still has a problem because of the large amount of computational time.

To solve this problem, many researchers have studied a magnetic equivalent circuit (MEC) model which is a good compromise between simplicity and accuracy [2], [3]. In spite of numerous studies about MEC such as leakage flux and magnetic saturation of various kinds of machines, cogging torque calculation of IPMSMs has been only conducted by the FEM [3].

In this paper, a new MEC method is proposed to calculate the air-gap flux density distribution and cogging torque waveform in IPMSM. Particularly, the proposed MEC method can consider the *slotting effect* by using conformal mapping and the *magnet end effect* based on the proposed Gaussian approximation equation.

Furthermore, the proposed MEC method is applied to the IPMSMs design. The objectives of the optimal design are to minimize the cogging torque and total harmonic distortion (THD) of back-electromotive force (back-EMF) and maximize the amplitude of back-EMF. The applicability of the proposed MEC method is verified by the practical motor design.

II. Cogging Torque Calculation Using Proposed MEC

The cogging torque, generated by interaction between the rotor magnets and stator teeth, can be calculated based on the Maxwell stress tensor as

$$T = \frac{L_{ef}}{\mu_o} \int_0^{2\pi} B_n B_t r^2 d\theta_m \quad (1)$$

where L_{ef} is the effective axial length, μ_o is the permeability of free air space, θ_m is the rotor position in mechanical angle, r is the radius of integration path, and B_n and B_t are the normal and the tangential air-gap flux densities, respectively.

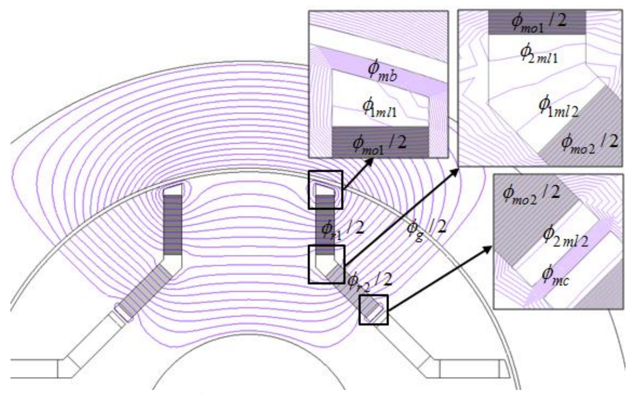

Fig. 1. Flux distribution of the IPMSM.

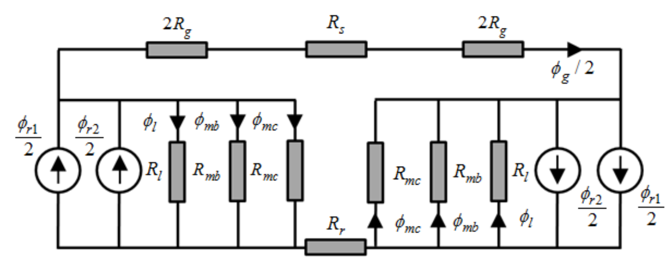

Fig. 2. MEC of the IPMSM.

A. Air-gap flux density analysis

Fig. 1 shows the flux distribution of the IPMSM (ϕ_{ri}: the flux sources over i^{th} magnet pole, ϕ_{gi}: the flux passing through the pole arc area, ϕ_{moi}: the leakage flux of permanent magnet over one magnet pole, ϕ_{kmli}: the k^{th} magnet end-leakage flux, and ϕ_{mb} and ϕ_{mc}: the leakage flux through the bridge and center post, respectively). In Fig. 2, the basic MEC model is constructed based on the position of the flux illustrated in Fig. 1.

R is the reluctance corresponding to each flux path, R_s and R_r are the stator and rotor reluctances, and R_l is leakage reluctance as follows:

$$R_l = 2R_{mo1} \,//\, R_1 ml_1 \,//\, R_1 ml_2 \,//\, 2R_{mo1} \,//\, R_2 ml_1 \,//\, R_2 ml_2 . \quad (2)$$

By using the basic MEC, the peak value of B_n can be calculated [2], [3] as shown in Fig 3.

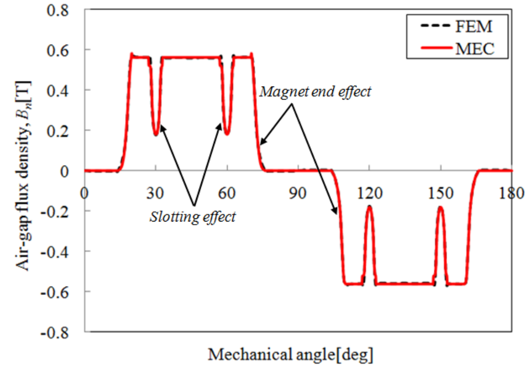

Fig. 3. Comparison of calculated B_n by using FEM and MEC.

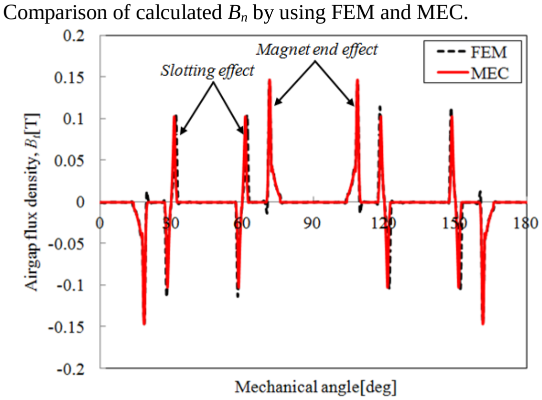

Fig. 4. Comparison of calculated B_t by using FEM and MEC.

B. Considering slotting effect

To take the *slotting effect* into account, conformal mapping, which has a robust characteristic on the variation of design parameters, is applied. The conformal mapping method based on the theory of complex variable functions is used to represent a bounded area in the plane of another complex variable. This method is powerful when solving magnetic field distributions between the equipotential boundaries like slots [3], [4]. The derivation of the conformal mapping will be presented in the full paper.

C. Considering magnet end effect

In this paper, the *magnet end effect* is defined as the increasing and decreasing the air-gap flux density on the magnet end of the rotor. To compute the *magnet end effect*, we propose a Gaussian approximation equation as follows:

$$f(\theta) = a\,e^{-\left(\frac{\theta - b}{c}\right)^2}. \tag{3}$$

where *a*, *b*, and *c* are the coefficients related with the rotor construction. The detail explanations about the coefficients will be given in the full paper.

III. Verification of Analyzed Air-Gap Flux Density and Cogging Torque

To verify the accuracy of the proposed MEC, the computed B_n, B_t and cogging torque by using the proposed method are compared with the analyzed data from the FEM. The flux density distributions calculated by the MEC and FEM are well matched as illustrated in Fig. 3 and 4. The cogging torque results calculated by two kinds of method are also fitted well with 3.23 % of the difference and 0.0146 of root mean square error (RMSE) as shown in Fig. 4 and TABLE I.

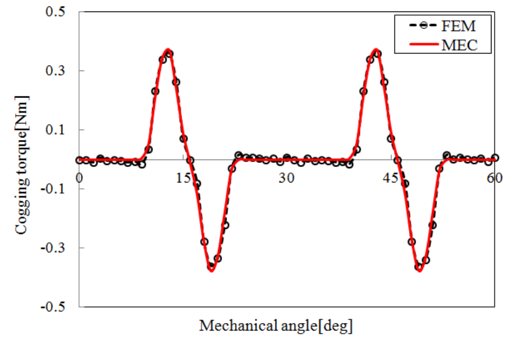

Fig. 5. Comparison of cogging torque waveform by using FEM and MEC.

TABLE I
Comparison Results of Cogging Torque by Using FEM and MEC

Cogging torque (FEM)	Cogging torque (Proposed MEC)	Difference	RMSE
0.372 Nm	0.360 Nm	3.23 %	0.0146

IV. Optimal IPMSM Design by Proposed MEC

In this section, the proposed MEC is applied in the IPMSM design to validate the feasibility. For the optimal design of IPMSM, various objectives and design variables should be taken into account [5]. If the optimal design has to be produced by using the FEM, too much computational time should be required. In this research, the optimal design to minimize the cogging torque and THD of back-EMF and maximize the amplitude of back-EMF is obtained with very short time.

V. Conclusion

The analysis and design of IPMSMs by using the FEM require large computational time. Hence, the most important meaning of this paper is the proposal of the analytical MEC method, which is the fast and accurate. Furthermore, the feasibility of the proposed method is verified by being applied to the practical design of IPMSM.

Due to the fact that the proposed MEC method can be applied to analyze other types of motors as well, this method could be widely used for the analysis and design of diverse types of motors.

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